Mathematics:

"A Story of Units"

Parent Handbook

Grade 5
Module 4

Grade 5 • Module 4

Multiplication and Division of Fractions and Decimal Fractions

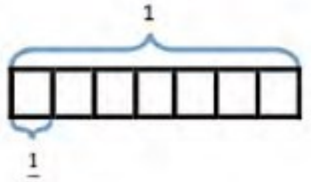
OVERVIEW

In Module 4, students learn to multiply fractions and decimal fractions and begin work with fraction division.

Topic A begins the 38-day module with an exploration of fractional measurement. Students construct line plots by measuring the same objects using three different rulers accurate to 1/2, 1/4, and 1/8 of an inch.

Students compare the line plots and explain how changing the accuracy of the unit of measure affects the distribution of points. This is foundational to the understanding that measurement is inherently imprecise, as it is limited by the accuracy of the tool at hand. Students use their knowledge of fraction operations to explore questions that arise from the plotted data. The interpretation of a fraction as division is inherent in this exploration. To measure to the quarter inch, one inch must be divided into four equal parts, or 1 ÷ 4. This reminder of the meaning of a fraction as a point on a number line, coupled with the embedded, informal exploration of fractions as division, provides a bridge to Topic B's more formal treatment of fractions as division.

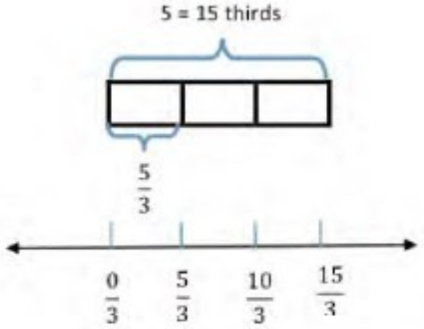
Interpreting fractions as division is the focus of Topic B. Equal sharing with area models (both concrete and pictorial) gives students an opportunity to make sense of division of whole numbers with answers in the form of fractions or mixed numbers (e.g., seven brownies shared by three girls; three pizzas shared by four people). Discussion also includes an interpretation of remainders as a fraction**.** Tape diagrams provide a linear model of these problems. Moreover, students see that by renaming larger units in terms of smaller units, division resulting in a fraction is just like whole number division.



1 week ÷ 7 = 7 days ÷ 7 = 1 day

1 ÷ 7 = 7 sevenths ÷ 7 = 1 seventh

1 ÷ 7 = 7/7 ÷ 7 = 1/7



15 thirds ÷ 3 = 5 thirds

5 ÷ 3 = 5/3

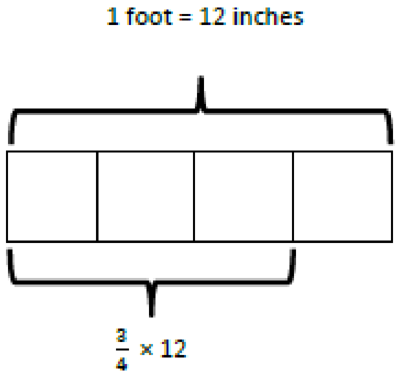
Topic B continues as students solve real world problems and generate story contexts for visual models. The topic concludes with students making connections between models and equations while reasoning about their results (e.g., between what two whole numbers does the answer lie?).

In Topic C, students interpret finding a fraction of a set (3/4 *of* 24) as multiplication of a whole number by a fraction (3/4 × 24) and use tape diagrams to support their understandings. This in turn leads students to see division by a whole number as equivalent to multiplication by its reciprocal. That is, division by 2, for example, is the same as multiplication by 1/2. Students also use the commutative property to relate a fraction of a set to the Grade 4 repeated addition interpretation of multiplication by a fraction. This opens the door for students to reason about various strategies for multiplying fractions and whole numbers. Students apply their knowledge of fraction of a set and previous conversion experiences (with scaffolding from a conversion chart, if necessary) to find a fraction of a measurement, thus converting a larger unit to an equivalent smaller unit (e.g.,1/3 min = 20 seconds and 2 1/4 feet = 27 inches).

Interpreting numerical expressions opens Topic D as students learn to evaluate expressions with parentheses, such as 3 × (2/3 –1/5) or 2/3 × (7 + 9). They then learn to interpret numerical expressions such as 3 *times the difference between 2/3 and 1/5* or *two-thirds the sum of 7 and 9*. Students generate word problems that lead to the same calculation, such as, “Kelly combined 7 ounces of carrot juice and 5 ounces of orange juice in a glass. Jack drank 2/3 of the mixture. How much did Jack drink?” Solving word problems allows students to apply new knowledge of fraction multiplication in context, and tape diagrams are used to model multi-step problems requiring the use of addition, subtraction, and multiplication of fractions.

Topic E introduces students to multiplication of fractions by fractions—both in fraction and decimal form. The topic starts with multiplying a unit fraction by a unit fraction, and progresses to multiplying two non-unit fractions. Students use area models, rectangular arrays, and tape diagrams to model the multiplication. These familiar models help students draw parallels between whole number and fraction multiplication, and solve word problems. This intensive work with fractions positions students to extend their previous work with decimal-by-whole number multiplication to decimal-by-decimal multiplication. Just as students used unit form to multiply fractional units by wholes in Module 2 (e.g., 3.5 × 2 = 35 tenths × 2 ones = 70 tenths), they will connect fraction-by-fraction multiplication to multiply fractional units-by-fractional units (3.5 × 0.2 = 35 tenths × 2 tenths = 70 hundredths). Reasoning about decimal placement is an integral part of these lessons. Finding fractional parts of customary measurements and measurement conversion concludes Topic E. Students convert smaller units to fractions of a larger unit (e.g., 6 inches = 1/2 ft). The inclusion of customary units provides a meaningful context for many common fractions (1/2 pint = 1 cup, 1/3 yard = 1 foot, 1/4 gallon = 1 quart, etc.). This topic, together with the fraction concepts and skills learned in Module 3, opens the door to a wide variety of application word problems.

$\frac{3}{4}$ of a foot = $\frac{3}{4}$ × 12 inches



Express $5\frac{3}{4}$ ft as inches.

$5\frac{3}{4}$ ft = (5 × 12) inches + ($\frac{3}{4}$ × 12) inches

= 60 + 9 inches

= 69 inches

Students interpret multiplication in Grade 3 as equal groups, and in Grade 4 students begin to understand multiplication as comparison. Here, in Topic F, students once again extend their understanding of multiplication to include scaling**.** Students compare the product to the size of one factor, given the size of the other factor without calculation (e.g., 486 × 1,327.45 is twice as large as 243 × 1,327.45, because 486 = 2 × 243). This reasoning, along with the other work of this module, sets the stage for students to reason about the size of products when quantities are multiplied by numbers larger than 1 and smaller than 1. Students relate their previous work with equivalent fractions to interpreting multiplication by *n/n* as multiplication by 1. Students build on their new understanding of fraction equivalence as multiplication by *n/n* to convert fractions to decimals and decimals to fractions. For example, 3/25 is easily renamed in hundredths as 12/100 using multiplication of 4/4. The word form of *twelve hundredths* will then be used to notate this quantity as a decimal. Conversions between fractional forms will be limited to fractions whose denominators are factors of 10, 100, or 1,000. Students will apply the concepts of the topic to real world, multi-step problems.

Topic G begins the work of division with fractions, both fractions and decimal fractions. Students use tape diagrams and number lines to reason about the division of a whole number by a unit fraction and a unit fraction by a whole number. Using the same thinking developed in Module 2 to divide whole numbers, students reason about how many *fourths* are in 5 when considering such cases as 5 ÷ 1/4. They also reason about the size of the unit when 1/4 is partitioned into 5 equal parts: 1/4 ÷ 5. Using this thinking as a backdrop, students are introduced to decimal fraction divisors and use equivalent fraction and place value thinking to reason about the size of quotients, calculate quotients, and sensibly place the decimal in quotients.

The module concludes with Topic H, in which numerical expressions involving fraction-by-fraction multiplication are interpreted and evaluated. Students create and solve word problems involving both multiplication and division of fractions and decimal fractions.

**The sample questions/responses contained in this manual are straight from http://www.engageny.org/. They are provided to give some insight into the kinds of skills expected of students as each lesson is taught.

Terminology

New or Recently Introduced Terms

- Decimal divisor (the number that divides the whole and that has units of tenths, hundredths, thousandths, etc.)
- Simplify (using the largest fractional unit possible to express an equivalent fraction)

Familiar Terms and Symbols

- Denominator (denotes the fractional unit, e.g., fifths in 3 fifths, which is abbreviated to the 5 in $\frac{3}{5}$)
- Decimal fraction
- Conversion factor
- Commutative Property (e.g., $4 \times \frac{1}{2} = \frac{1}{2} \times 4$)
- Distribute (with reference to the distributive property, e.g., in $1\frac{2}{5} \times 15 = (1 \times 15) + (\frac{2}{5} \times 15)$)
- Divide, division (partitioning a total into equal groups to show how many units in a whole, e.g., $5 \div \frac{1}{5} = 25$)
- Equation (statement that two expressions are equal, e.g., $3 \times 4 = 6 \times 2$)
- Equivalent fraction
- Expression
- Factors (numbers that are multiplied to obtain a product)
- Feet, mile, yard, inch, gallon, quart, pint, cup, pound, ounce, hour, minute, second
- Fraction greater than or equal to 1 (e.g., $\frac{7}{2}$, $3\frac{1}{2}$, an abbreviation for $3 + \frac{1}{2}$)
- Fraction written in the largest possible unit (e.g., $\frac{3}{6} = \frac{1 \times 3}{2 \times 3} = \frac{1}{2}$ or 1 three out of 2 threes $= \frac{1}{2}$)
- Fractional unit (e.g., the fifth unit in 3 fifths denoted by the denominator 5 in $\frac{3}{5}$)
- Hundredth ($\frac{1}{100}$ or 0.01)
- Line plot
- Mixed number ($3\frac{1}{2}$, an abbreviation for $3 + \frac{1}{2}$)
- Numerator (denotes the count of fractional units, e.g., 3 in 3 fifths or 3 in $\frac{3}{5}$)
- Parentheses (symbols ( ) used around a fact or numbers within an equation)
- Quotient (the answer when one number is divided by another)
- Tape diagram (method for modeling problems)

- Tenth ($\frac{1}{10}$ or 0.1)
- Unit (one segment of a partitioned tape diagram)
- Unknown (the missing factor or quantity in multiplication or division)
- Whole unit (any unit that is partitioned into smaller, equally sized fractional units)

Suggested Tools and Representations

- Area models
- Number lines
- Tape diagrams

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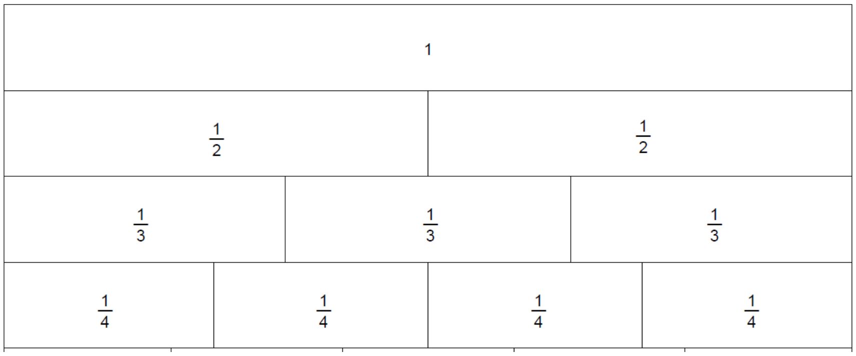
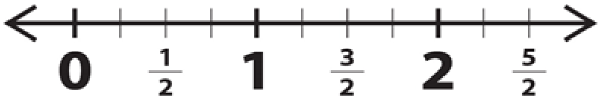
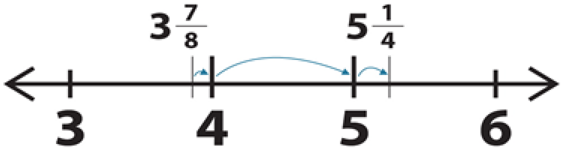
Fraction Strips: Fraction strips are tiles or strips that are proportionately sized to one whole so that students may physically make size comparisons and find equivalent amounts using different denominators.



Number Lines: The number line is showing fractions and improper fractions located between whole numbers. The "empty" number line initially shows whole numbers only, without portions or increments between whole numbers indicated, so as to allow for students to approximate locations of fractions, improper fractions and/or mixed numbers (ie. 5¼).

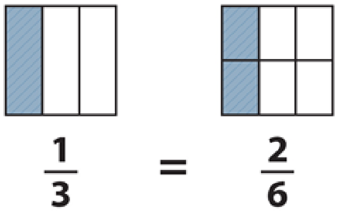


Example of a number line



Example of an "empty" number line

Rectangular Fractional Model: This rectangular fraction model allows students to begin with two whole rectangles of equal size which can be broken into thirds and sixths in order to find equivalent portions for fractions with unlike denominators.

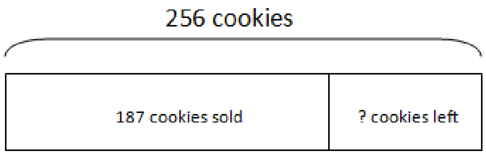


Example of a rectangular fraction model

Tape Diagram: Tape diagrams, also called bar models, are pictorial representations of relationships between quantities used to solve word problems. At the heart of a tape diagram is the idea of *forming units*. In fact, forming units to solve word problems is one of the most powerful examples of the unit theme and is particularly helpful for understanding fraction arithmetic. The tape diagram provides an essential bridge to algebra and is often called “pictorial algebra.” There are two basic forms of the tape diagram model. The first form is sometimes called the part-whole model; it uses bar segments placed end-to-end (Grade 3 Example), while the second form, sometimes called the comparison model, uses two or more bars stacked in rows that are typically left justified. (Grade 5 Example depicts this model.)

Grade 3 Example: *Sarah baked 256 cookies. She sold some of them. 187 were left. How many did she sell?*

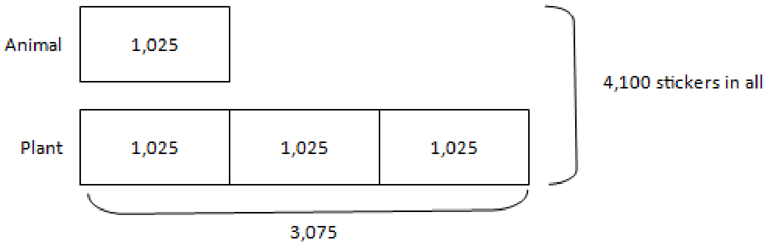
256 – 187 = _____



256 –187 = 69

Sarah sold 69 cookies.

Grade 5 Example: *Sam has 1,025 animal stickers. He has 3 times as many plant stickers as animal stickers. How many plant stickers does Sam have? How many stickers does Sam have altogether?*

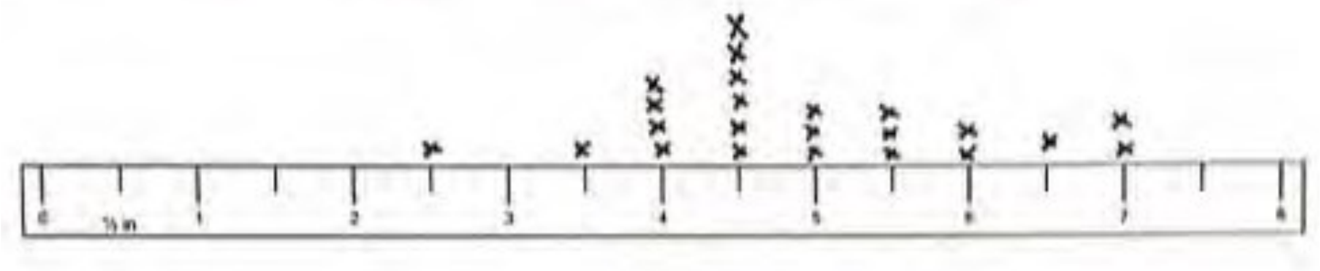


1. He has 3,075 plant stickers.
2. He has 4,100 stickers altogether.

Lesson 1

Objective: Measure and compare pencil lengths to the nearest 1/2, 1/4, and 1/8 of an inch and analyze the data through line plots.

2. Using a ruler, measure your pencil strip to the nearest $\frac{1}{2}$ inch and mark the measurement with an X above the ruler below. Construct a line plot of your classmates' pencil measurements.

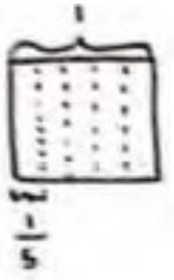


Lesson 2

Objective: Interpret a fraction as division.

1. Draw a picture to show the division. Write a division expression using unit form. Then express your answer as a fraction. The first one is done for you.

   a. 1 ÷ 5 = 5 fifths ÷ 5 = 1 fifth = $\frac{1}{5}$



4. Fill in the blanks to make true number sentences.

   a. 2 ÷ 3 = $\frac{2}{3}$

   b. 15 ÷ 8 = $\frac{15}{8}$

   c. 11 ÷ 4 = $\frac{11}{4}$

Lesson 3

Objective: Interpret a fraction as division.

1. Fill in the chart. The first one is done for you.

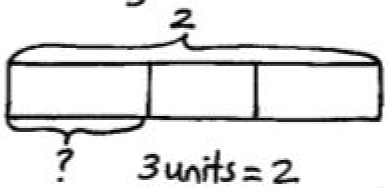
Division Expression	Unit Forms	Improper Fraction	Mixed Numbers	Standard Algorithm (Write your answer in whole numbers and fractional units, then check.)
a. 5 ÷ 4	20 fourths ÷ 4 = 5 fourths	$\frac{5}{4}$	$1\frac{1}{4}$	$\begin{array}{r} 1\frac{1}{4} \\ 4\overline{)\,5} \\ -4 \\ \hline 1 \end{array}$ Check $4 \times 1\frac{1}{4} = 1\frac{1}{4} + 1\frac{1}{4} + 1\frac{1}{4} + 1\frac{1}{4}$ $= 4 + \frac{4}{4}$ $= 4 + 1$ $= 5$
b. 3 ÷ 2	6 halves ÷ 2 = 3 halves	$\frac{3}{2}$	$1\frac{1}{2}$	$\begin{array}{r} 1\frac{1}{2} \\ 2\overline{)\,3} \\ -2 \\ \hline 1 \end{array}$ $2 \times 1\frac{1}{2} = 1\frac{1}{2} + 1\frac{1}{2}$ $= 2 + \frac{2}{2}$ $= 2 + 1$ $= 3$

Lesson 4

Objective: Use tape diagrams to model fractions as division.

1. Draw a tape diagram to solve. Express your answer as a fraction. Show the multiplication sentence to check your answer. The first one is done for you.

b. $2 \div 3 = \frac{2}{3}$



3 units = 2

1 unit = $2 \div 3 = \frac{2}{3}$

$$\begin{array}{r} 0\frac{2}{3} \\ 3\overline{)\,2} \\ -0 \\ \hline 2 \end{array}$$

Check: $3 \times \frac{2}{3}$

$= \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$

$= \frac{6}{3}$

$= 2$

2. Fill in the chart. The first one is done for you.

Division expression	Fraction	Between what two whole numbers is your answer?	Standard algorithm
d. 32 ÷ 40	$\frac{32}{40}$	0 and 1	$\begin{array}{r} 0\frac{32}{40} \\ 40\overline{)\,32} \\ -0 \\ \hline 32 \end{array}$

Lesson 5

Objective: Solve word problems involving the division of whole numbers with answers in the form of fractions or whole numbers.

Lillian had 2 two-liter bottles of soda, which she distributed equally between 10 glasses.

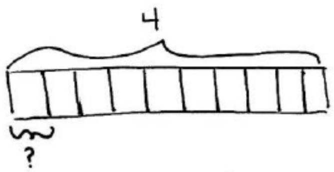
a. How much soda was in each glass? Express your answer as a fraction of a liter.

b. Express your answer as a decimal number of liters.

c. Express your answer as a whole number of milliliters.

2 TWO-LITERS IN 10 GLASSES

2 TWO-LITERS = 4 LITERS



10 UNITS = 4 LITERS

1 UNIT = 4 ÷ 10

? = $\frac{4}{10}$ LITERS

a) EACH GLASS WILL HAVE $\frac{4}{10}$ LITERS OF SODA.

b) $\frac{4}{10}$ = 4 TENTHS

= 0.4

EACH GLASS WILL HAVE 0.4 LITERS OF SODA.

c) 1 LITER = 1,000 mL

0.4 x 1,000 = 400

0.4 L = 400 mL

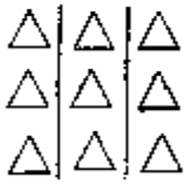
EACH GLASS WILL HAVE 400 mL OF SODA.

Lesson 6

Objective: Relate fractions as division to fraction of a set.

1. Find the value of each of the following.

a.

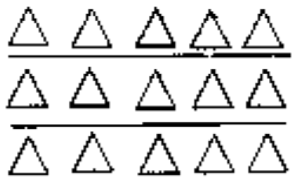


$\frac{1}{3}$ of 9 = 3

$\frac{2}{3}$ of 9 = 6

$\frac{3}{3}$ of 9 = 9

b.



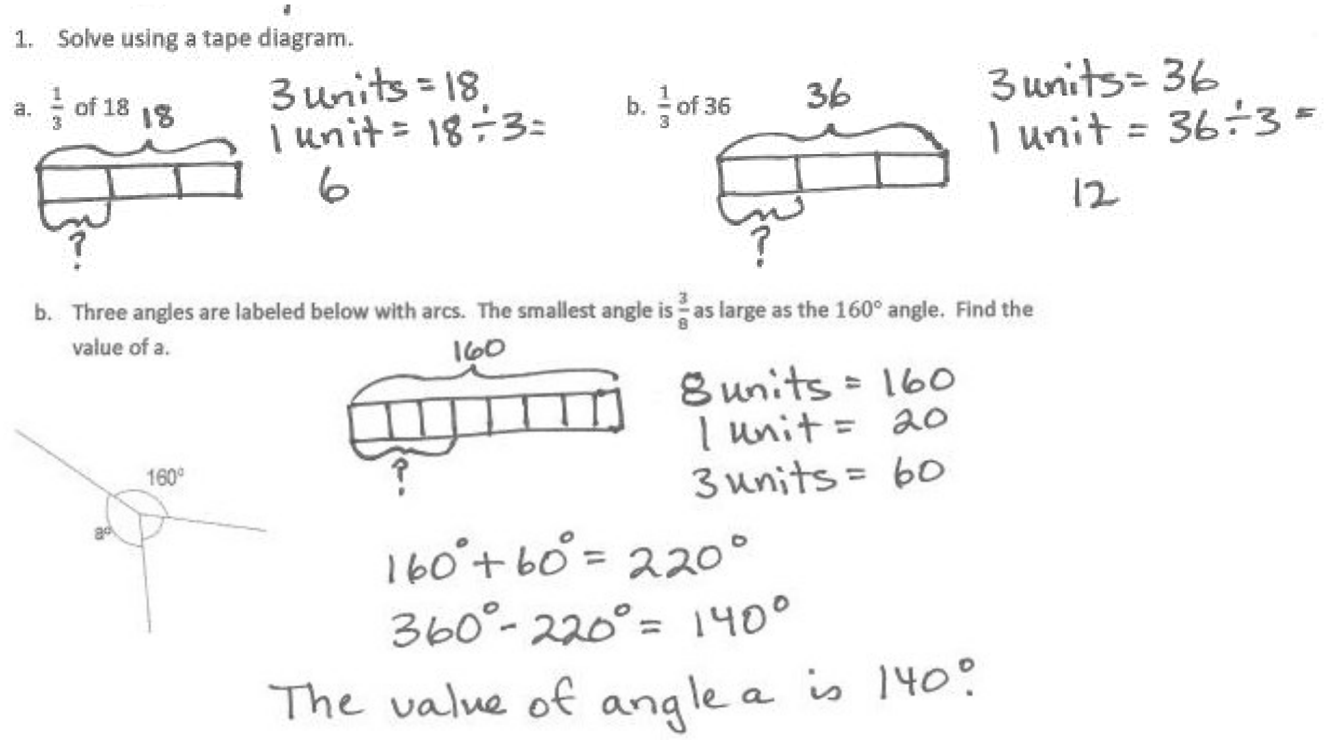
$\frac{1}{3}$ of 15 = 5

$\frac{2}{3}$ of 15 = 10

$\frac{3}{3}$ of 15 = 15

Lesson 7

Objective: Multiply any whole number by a fraction using tape diagrams.



1. Solve using a tape diagram.

a. $\frac{1}{3}$ of 18

18

?

3 units = 18
1 unit = 18 ÷ 3 =
6

b. $\frac{1}{3}$ of 36

36

?

3 units = 36
1 unit = 36 ÷ 3 =
12

b. Three angles are labeled below with arcs. The smallest angle is $\frac{3}{8}$ as large as the 160° angle. Find the value of a.

160°

a°

160

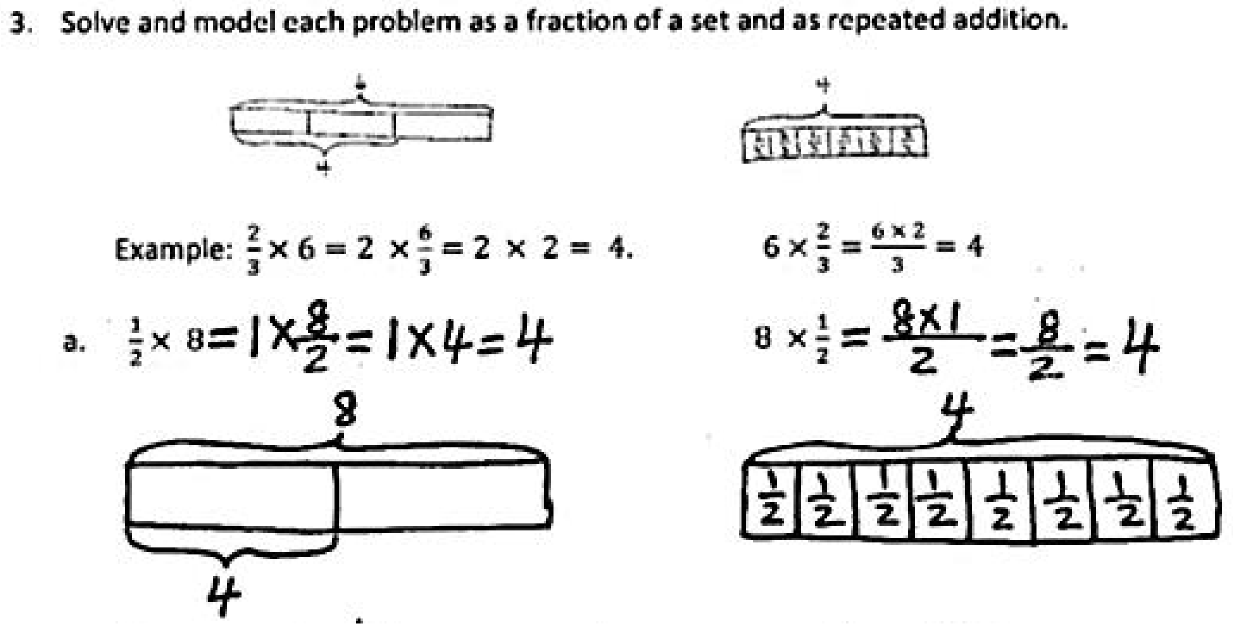
?

8 units = 160
1 unit = 20
3 units = 60

160° + 60° = 220°
360° - 220° = 140°
The value of angle a is 140°.

Lesson 8

Objective: Relate fraction of a set to the repeated addition interpretation of fraction multiplication.



3. Solve and model each problem as a fraction of a set and as repeated addition.

4

4

4

Example: $\frac{2}{3} \times 6 = 2 \times \frac{6}{3} = 2 \times 2 = 4.$

$6 \times \frac{2}{3} = \frac{6 \times 2}{3} = 4$

a. $\frac{1}{2} \times 8 = 1 \times \frac{8}{2} = 1 \times 4 = 4$

$8 \times \frac{1}{2} = \frac{8 \times 1}{2} = \frac{8}{2} = 4$

8

4

4

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

Lesson 9

Objective: Find a fraction of a measurement, and solve word problems.

1. Convert. Show your work using a tape diagram or an equation. The first one is done for you.

c. $\frac{5}{6}$ year = ________ months

$\frac{5}{6}$ year $= \frac{5}{6} \times 1$ year

$= \frac{5}{6} \times 12$ months

$= \frac{60}{6}$ months

$= 10$ months

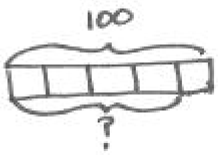
d. $\frac{4}{5}$ meter = ________ centimeters

$\frac{4}{5}$ m $= \frac{4}{5} \times 1$ m

$= \frac{4}{5} \times 100$ cm

$= \frac{400}{5}$ cm

$= 80$ cm



5 units = 100
1 unit = 20
4 units = 80cm

2. Mrs. Lang told her class that their class pet hamster is $\frac{1}{4}$ ft in length. How long is the hamster in inches?

$\frac{1}{4}$ ft $= \frac{1}{4} \times 1$ ft $= \frac{1}{4} \times 12$ in $= \frac{12}{4}$ inches $= 3$ inches

The hampster is 3 inches long.

Lesson 10

Objective: Compare and evaluate expressions with parentheses.

2. Write an expression to match, then evaluate.

a. $\frac{1}{6}$ the sum of 16 and 20.

$(16+20) \times \frac{1}{6} =$

$36 \times \frac{1}{6} =$

$\frac{36 \times 1}{6} = 6$

4. Use <, >, or = to make true number sentences without calculating. Explain your thinking.

a. $4 \times 2 + 4 \times \frac{2}{3}$ $\bigcirc\!\!\!\!>$ $3 \times \frac{2}{3}$

$4 \times \frac{2}{3}$ is more than $3 \times \frac{2}{3}$ without adding (4×2).

b. $\left(5 \times \frac{3}{4}\right) \times \frac{2}{5}$ $\bigcirc\!\!\!\!>$ $\left(5 \times \frac{3}{4}\right) \times \frac{2}{7}$

$\frac{2}{5}$ of a number is more than $\frac{2}{7}$ of the same number

Lesson 11

Objective: Solve and create fraction word problems involving addition, subtraction, and multiplication.

3. Jack, Jill, and Bill each carried a 48-ounce bucket full of water down the hill. By the time they reached the bottom, Jack's bucket was only $\frac{3}{4}$ full, Jill's was $\frac{2}{3}$ full, and Bill's was $\frac{1}{6}$ full. How much water did they spill altogether on their way down the hill?

Jack $\frac{1}{4} \times 48 = \frac{48}{4} = 12$ oz

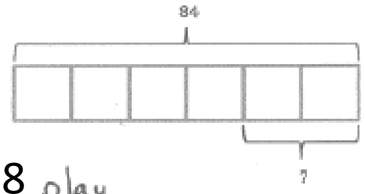
Jill $\frac{1}{3} \times 48 = \frac{48}{3} = 16$ oz

Bill $\frac{5}{6} \times 48 = \frac{240}{6} = 40$ oz

$12 + 16 + 40 = 68$ oz

Together they spilt 68 oz. of water.

Create a story problem based on the tape diagram and numbers used below. Your story must include a fraction.



28 play the violin.

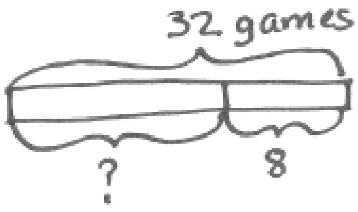
Two sixths of an orchestra play the violin. How many Musicians play the violin?

$\frac{2}{6}$ of $84 = \frac{1}{3}$ of $84 = \frac{84}{3} = 28$

Lesson 12

Objective: Solve and create fraction word problems involving addition, subtraction, and multiplication.

1. A baseball team played 32 games, and lost 8. Katy was the catcher in $\frac{5}{8}$ of the winning games and $\frac{1}{4}$ of the losing games.
   a. What fraction of the games did the team win?



$32 - 8 = 24$ $\frac{24}{32} = \frac{3}{4}$

The team won $\frac{3}{4}$ of the games they played.

   b. How many games did Katy play catcher?

Katy won: $\frac{5}{8} \times 24 = \frac{5 \times \cancel{24}^{3}}{\cancel{8}_{1}} = 15$ games

Lost: $\frac{1}{4} \times 8 = \frac{8}{4} = 2$ games

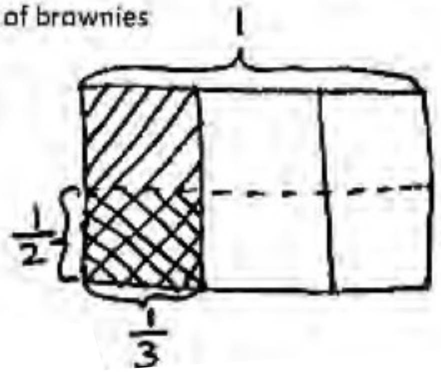
Katy played 17 games.

Lesson 13

Objective: Multiply unit fractions by unit fractions.

1. Solve. Draw an area model to show your thinking. Then write a multiplication sentence. The first one has been done for you.

b. Half of $\frac{1}{3}$ pan of brownies = $\frac{1}{6}$ pan of brownies

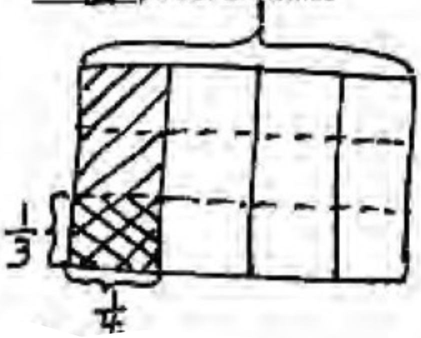


$\frac{1}{2}$ of $\frac{1}{3}$

$= \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{6}$

c. A fourth of $\frac{1}{3}$ pan of brownies = $\frac{1}{12}$ pan of brownies



$\frac{1}{4}$ of $\frac{1}{3}$

$= \frac{1}{4} \times \frac{1}{3}$

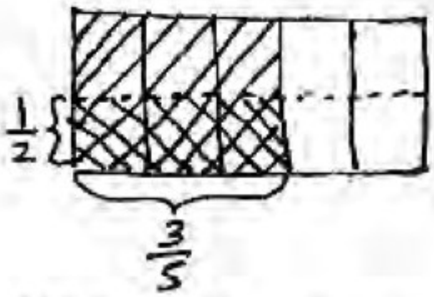
$= \frac{1}{12}$

Lesson 14

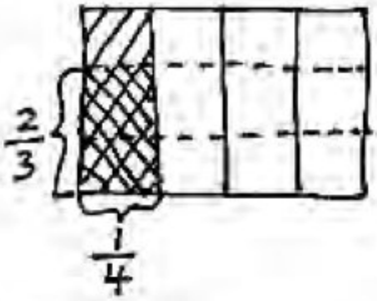
Objective: Multiply unit fractions by non-unit fractions.

1. Solve. Draw a model to explain your thinking. Then write a number sentence. An example has been done for you.

e. $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$



f. $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$

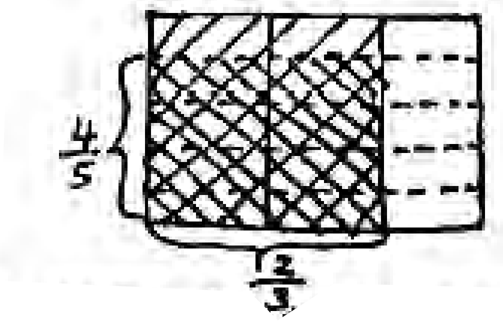


Lesson 15

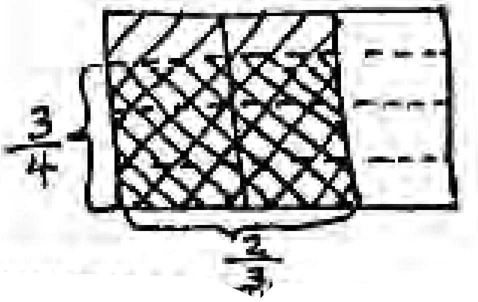
Objective: Multiply non-unit fractions by non-unit fractions.

1. Solve. Draw any model to explain your thinking. Then write a multiplication sentence. The first one is done for you.

d. $\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$



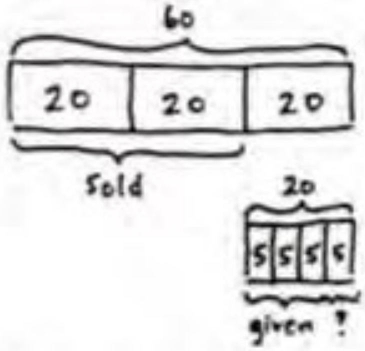
e. $\frac{3}{4} \times \frac{2}{3} = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}} \times \underset{1}{\cancel{3}}} = \frac{1}{2}$



Lesson 16

Objective: Solve word problems using tape diagrams and fraction-by-fraction multiplication.

1. Mrs. Onusko made 60 cookies for a bake sale. She sold $\frac{2}{3}$ of them and gave $\frac{3}{4}$ of the remaining cookies to the students working at the sale. How many cookies did she have left?



3units = 60 cookies
1unit = 20 cookies

20 ÷ 4 = 5 cookies

Mrs Onusko had 5 cookies left.

Lesson 17

Objective: Relate decimal and fraction multiplication.

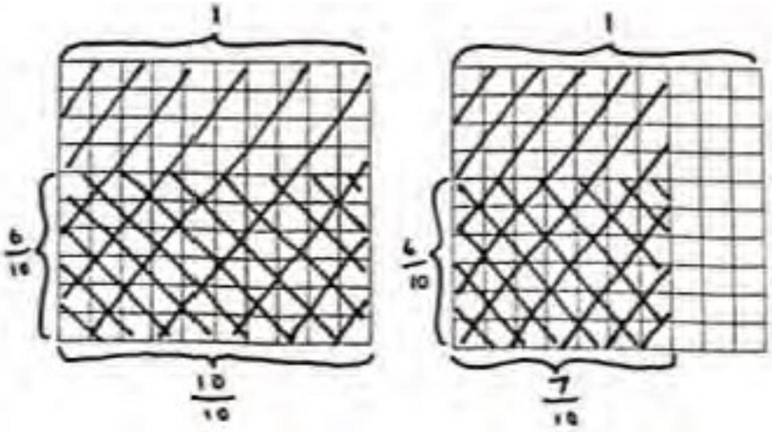
1. Multiply and model. Rewrite each expression as a multiplication sentence with decimal factors. The first one is done for you.

d. $\frac{6}{10} \times 1.7$

$= \frac{6}{10} \times \frac{17}{10}$

$= \frac{6 \times 17}{10 \times 10}$

$= \frac{102}{100} = 1\frac{2}{100}$



$0.6 \times 1.7 = 1.02$

Lesson 18

Objective: Relate decimal and fraction multiplication.

c. 6.06 × 2.8 =

$= \frac{606}{100} \times \frac{28}{10}$

$= \frac{606 \times 28}{1000}$

$= \frac{16968}{1000}$

$= 16.968$

$$\begin{array}{rl} 606 & \text{(hundredths)} \\ \times\quad 28 & \text{(tenths)} \\ \hline 4848 & \\ +12120 & \\ \hline 16968 & \text{(thousandths)} \end{array}$$

d. 3.3 × 0.14 =

$= \frac{33}{10} \times \frac{14}{100}$

$= \frac{33 \times 14}{1000}$

$= \frac{462}{1000}$

$= 0.462$

$$\begin{array}{rl} 33 & \text{(tenths)} \\ \times\quad 14 & \text{(hundredths)} \\ \hline 132 & \\ +330 & \\ \hline 462 & \text{(thousandths)} \end{array}$$

3. Solve using the standard algorithm. Use the thought bubble to show your thinking about the units of your product.

c. 8.31 × 2.4 = 19.944

$$\begin{array}{r} 8.31 \\ \times\quad 2.4 \\ \hline 3324 \\ +16620 \\ \hline 19.944 \end{array}$$

$$\begin{array}{rl} 831 & \text{(hundredths)} \\ \times\quad 24 & \text{(tenths)} \\ \hline 3324 & \\ +16620 & \\ \hline 19944 & \text{(thousandths)} \end{array}$$

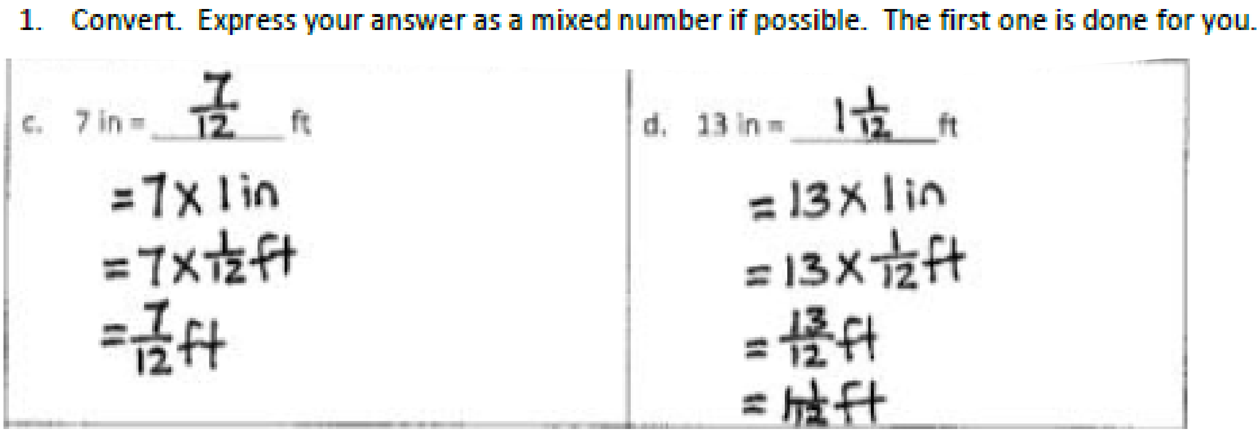
d. 7.50 × 3.5 = 26.25

$$\begin{array}{r} 7.50 \\ \times\quad 3.5 \\ \hline 3750 \\ +22500 \\ \hline 26.250 \end{array}$$

$$\begin{array}{rl} 750 & \text{(hundredths)} \\ \times\quad 35 & \text{(tenths)} \\ \hline 3750 & \\ +22500 & \\ \hline 26250 & \text{(thousandths)} \end{array}$$

Lesson 19

Objective: Convert measures involving whole numbers, and solve multi-step word problems.



1. Convert. Express your answer as a mixed number if possible. The first one is done for you.

c. 7 in = $\frac{7}{12}$ ft

$= 7 \times 1$ in

$= 7 \times \frac{1}{12}$ ft

$= \frac{7}{12}$ ft

d. 13 in = $1\frac{1}{12}$ ft

$= 13 \times 1$ in

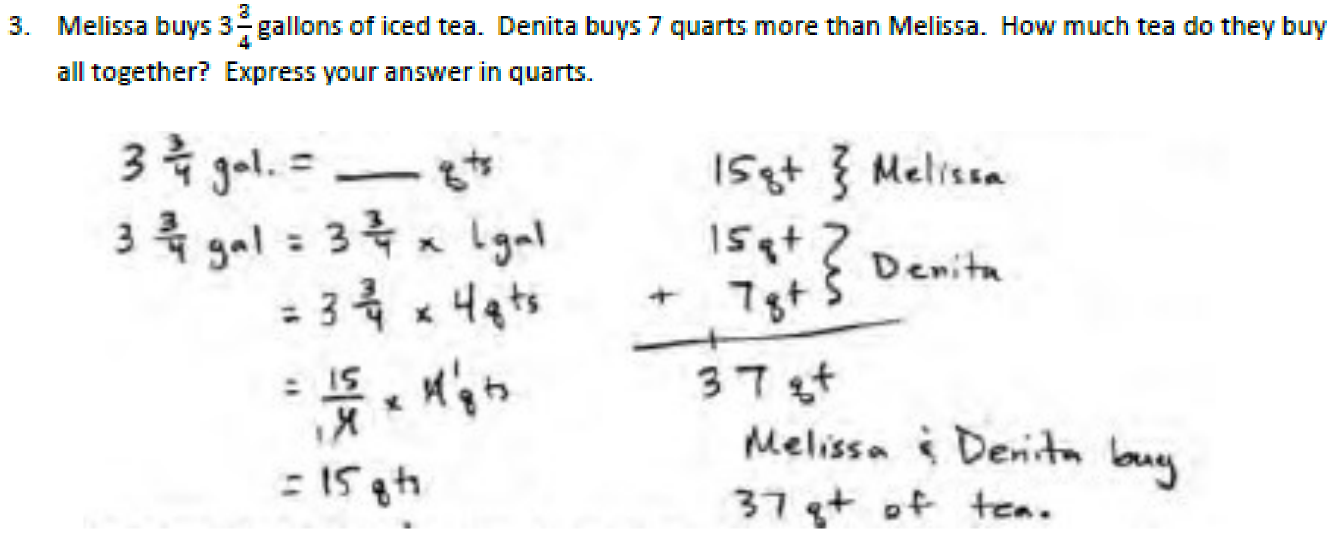
$= 13 \times \frac{1}{12}$ ft

$= \frac{13}{12}$ ft

$= 1\frac{1}{12}$ ft

Lesson 20

Objective: Convert mixed unit measurements, and solve multi-step word problems.



3. Melissa buys $3\frac{3}{4}$ gallons of iced tea. Denita buys 7 quarts more than Melissa. How much tea do they buy all together? Express your answer in quarts.

$3\frac{3}{4}$ gal. = ___ qts

$3\frac{3}{4}$ gal $= 3\frac{3}{4} \times 1$ gal

$= 3\frac{3}{4} \times 4$ qts

$= \frac{15}{{}_1\cancel{4}} \times \overset{1}{\cancel{4}}$ qts

$= 15$ qts

15 qt } Melissa

15 qt
+ 7 qt } Denita

$\overset{1}{3}7$ qt

Melissa & Denita buy 37 qt of tea.

Lesson 21

Objective: Explain the size of the product, and relate fraction and decimal equivalence to multiplying a fraction by 1.

2. Express each fraction as an equivalent decimal.

a. $\frac{1}{4} \times \frac{25}{25} = \frac{25}{100}$

$= 0.25$

b. $\frac{3}{4} \times \frac{25}{25} = \frac{75}{100}$

$= 0.75$

3. Jack said that if you take a number and multiply it by a fraction, the product will always be smaller than what you started with. Is he correct? Why or why not? Explain your answer and give at least two examples to support your thinking.

Jack is right some of the time, but not always. If you multiply by a fraction equal to 1 (like $\frac{2}{2}$) the product is = to what you started with.

ex 1.

$3 \times \frac{2}{2} = \frac{6}{2} = 3$

same #

ex 2.

$3 \times \frac{4}{4} = \frac{12}{4} = 3$

same #

Lesson 22

Objective: Compare the size of the product to the size of the factors.

1. Solve for the unknown. Rewrite each phrase as a multiplication sentence. Circle the scaling factor and put a box around the number of meters.

a. $\frac{1}{2}$ as long as 8 meters = 4 meters

$\enclose{circle}{\frac{1}{2}} \times \boxed{8} = 4$

b. 8 times as long as $\frac{1}{2}$ meter = 4 meters

$\enclose{circle}{8} \times \boxed{\frac{1}{2}} = 4$

4. Look at the inequalities in each box. Choose a single fraction to write in all three blanks that would make all three number sentences true. Explain how you know.

a. $\frac{3}{4} \times \underline{5/2} > \frac{3}{4}$ $\qquad 2 \times \underline{5/2} > 2$ $\qquad \frac{7}{5} \times \underline{5/2} > \frac{7}{5}$

Multiplying by a fraction greater than 1 will make the product larger than the other factor.

Lesson 23

Objective: Compare the size of the product to the size of the factors.

2.

a. Sort the following expressions by rewriting them in the table.

The product is less than the boxed number	The product is greater than the boxed number
0.3 x 0.069 602 x 0.489 0.2 x 0.1	13.89 x 1.004 0.72 x 1.24 102.03 x 4.015

$\boxed{13.89} \times 1.004$ $\boxed{602} \times 0.489$ $\boxed{102.03} \times 4.015$

$\boxed{0.3} \times 0.069$ $\boxed{0.72} \times 1.24$ $\boxed{0.2} \times 0.1$

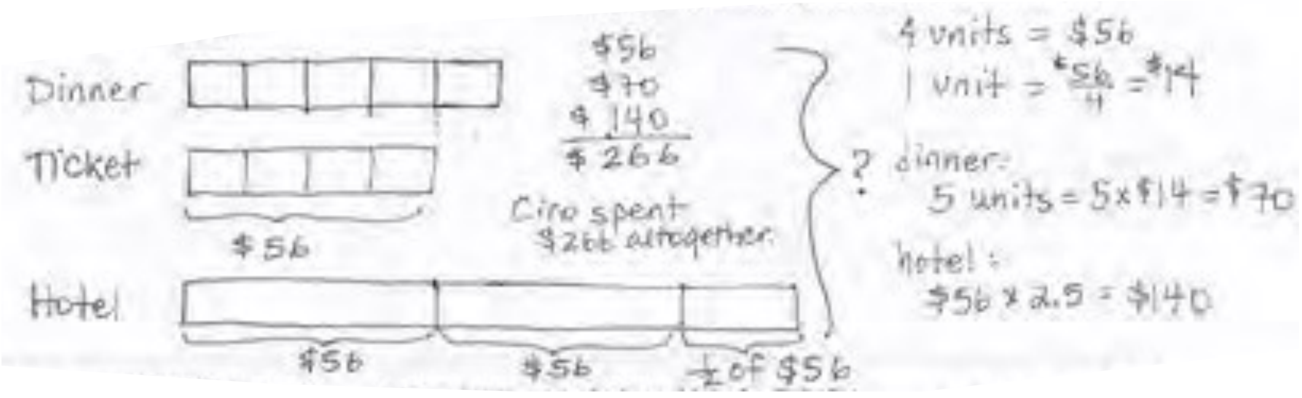
b. Explain your sorting by writing a sentence that tells what the expressions in each column of the table have in common.

In the first column, the boxed number is multiplied by a scaling factor less than 1, so the products will be less than the boxed number.
In the second column, the boxed number is multiplied by a scaling factor greater than 1.

Lesson 24

Objective: Solve word problems using fraction and decimal multiplication.

6. Ciro purchased a concert ticket for \$56. The cost of the ticket was $\frac{4}{5}$ the cost of his dinner. The cost of his hotel was $2\frac{1}{2}$ times as much as his ticket. How much did Ciro spend altogether for the concert ticket, hotel, and dinner?



Lesson 25

Objective: Divide a whole number by a unit fraction.

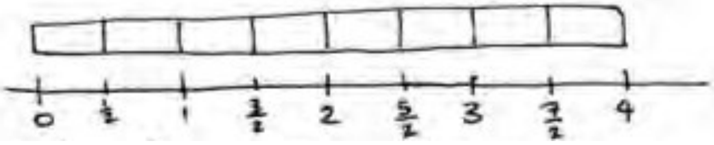
1. Draw a tape diagram and a number line to solve. You may draw the model that makes the most sense to you. Fill in the blanks that follow. Use the example to help you.

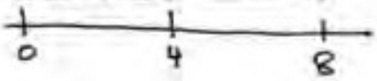
a. $4 \div \frac{1}{2} =$ 8

There are 2 halves in 1 whole.

There are 8 halves in 4 wholes.

If 4 is $\frac{1}{2}$ what is the whole? 8





2. Divide. Then multiply to check.

e. $2 \div \frac{1}{8}$	f. $7 \div \frac{1}{6}$	g. $8 \div \frac{1}{3}$	h. $9 \div \frac{1}{4}$
$2 \div \frac{1}{8} = 2 \times 8 = 16$ $\frac{1}{8} \times 16 = \frac{16}{8} = 2$ ✓	$7 \div \frac{1}{6} = 7 \times 6$ $= 42$ $\frac{1}{6} \times 42 = \frac{42}{6} = 7$ ✓	$8 \div \frac{1}{3} = 8 \times 3$ $= 24$ $\frac{1}{3} \times 24 = \frac{24}{3} = 8$ ✓	$9 \div \frac{1}{4} = 9 \times 4$ $= 36$ $\frac{1}{4} \times 36 = \frac{36}{4} = 9$ ✓

Lesson 26

Objective: Divide a unit fraction by a whole number.

1. Draw a model or tape diagram to solve. Use the thought bubble to show your thinking. Write your quotient in the blank. Use the example to help you.

Example: $\frac{1}{2} \div 3$

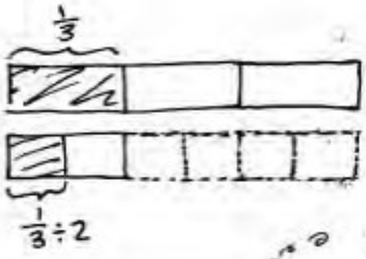
1 half ÷ 3

= 3 sixths ÷ 3

= 1 sixth

$\frac{1}{2} \div 3 = \frac{1}{6}$

a. $\frac{1}{3} \div 2 =$ $\frac{1}{6}$



1 third ÷ 2

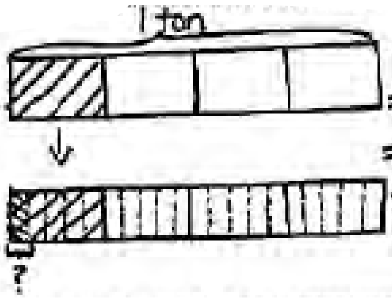
= 2 sixths ÷ 2

= 1 sixth

Lesson 27

Objective: Solve problems involving fraction division.

4. A pallet holding 5 identical crates weighs $\frac{1}{4}$ ton.
   a. How many tons does each crate weigh? Draw a picture to support your response.



$\frac{1}{4} \div 5$

= 1 fourth ÷ 5

= 5 twentieths ÷ 5

= 1 twentieth

Each crate weighs $\frac{1}{20}$ ton.

   b. How many pounds does each crate weigh?

$\frac{1}{20}$ ton = ___ pounds

$= \frac{1}{\cancel{20}_{1}} \times \frac{\overset{100}{\cancel{2000}}}{1}$

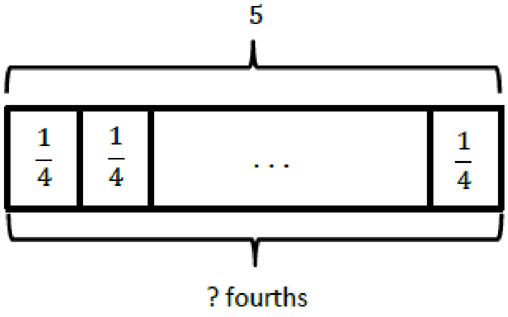
= 100 pounds

Each crate weighs 100 Pounds.

Lesson 28

Objective: Write equations and word problems corresponding to tape and number line diagrams.

1. Create and solve a division story problem about 5 meters of rope that is modeled by the tape diagram below.



$5 \div \frac{1}{4} = 20$

He will have 20 fourths.

Diego has 5 meters of rope. He cuts each meter equally into fourths. How many fourths will he have altogether?

3. Draw a tape diagram and create a word problem for the following expressions, and then solve.
   a. $2 \div \frac{1}{3}$

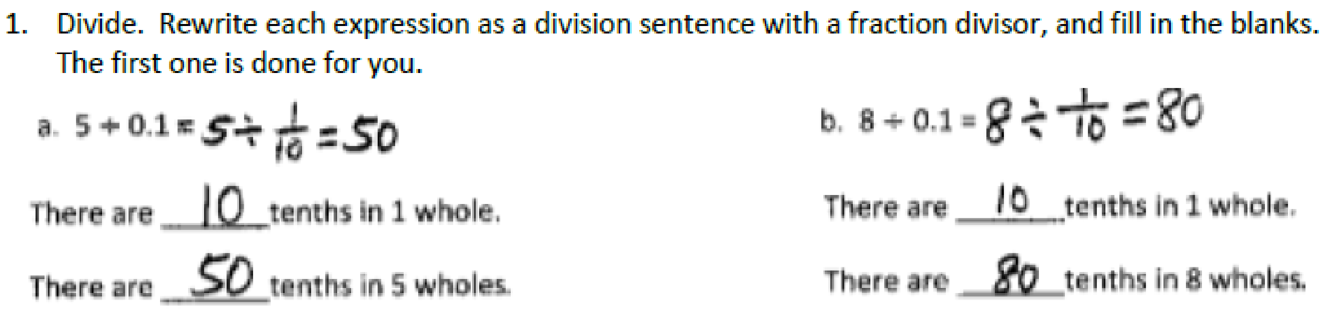
= 6

She will have 6

Phung bought 2 pizzas. She wants to cut each pizza into thirds. How many slices will she have altogether?

Lesson 29

Objective: Connect division by a unit fraction to division by 1 tenth and 1 hundredth.



1. Divide. Rewrite each expression as a division sentence with a fraction divisor, and fill in the blanks. The first one is done for you.

a. $5 \div 0.1 = 5 \div \frac{1}{10} = 50$

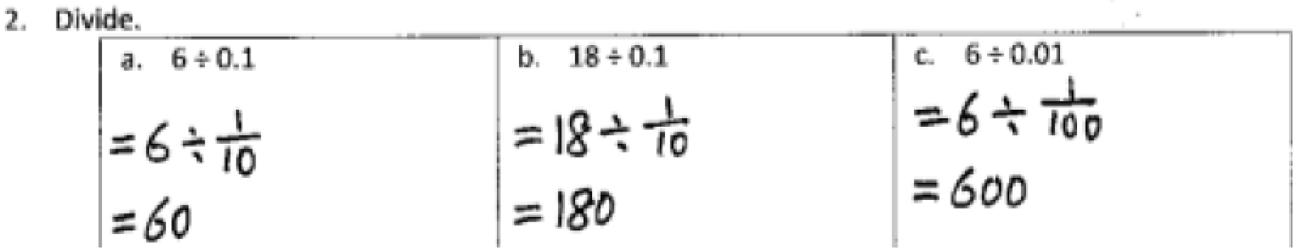
There are 10 tenths in 1 whole.

There are 50 tenths in 5 wholes.

b. $8 \div 0.1 = 8 \div \frac{1}{10} = 80$

There are 10 tenths in 1 whole.

There are 80 tenths in 8 wholes.

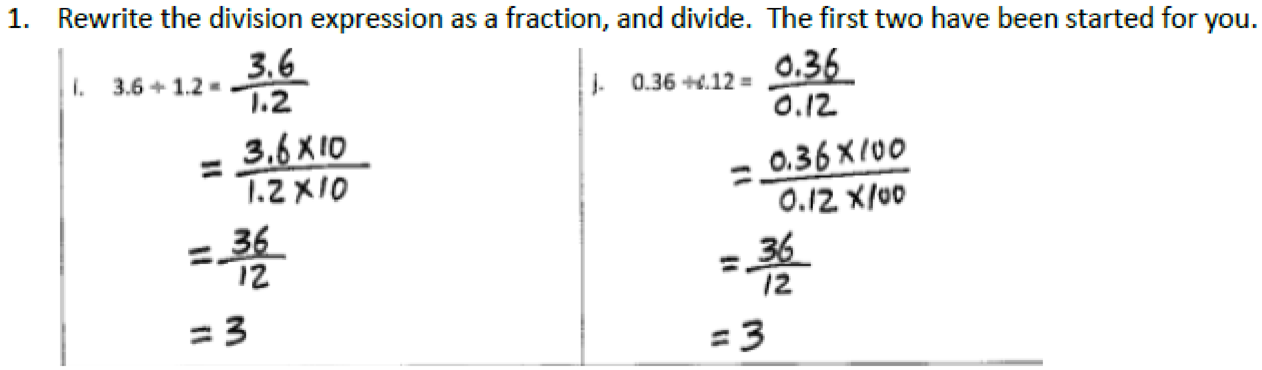


2. Divide.

a. $6 \div 0.1$	b. $18 \div 0.1$	c. $6 \div 0.01$
$= 6 \div \frac{1}{10}$ $= 60$	$= 18 \div \frac{1}{10}$ $= 180$	$= 6 \div \frac{1}{100}$ $= 600$

Lesson 30

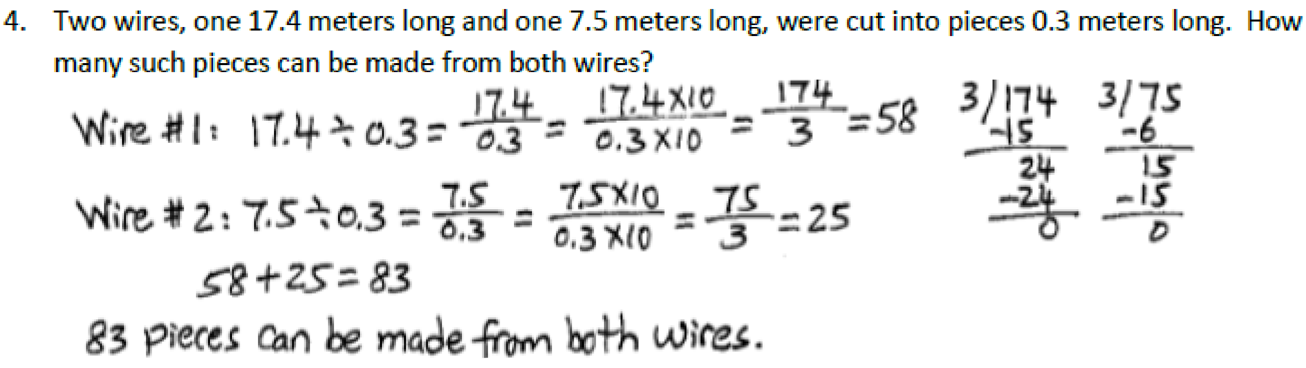
Objective: Divide decimal dividends by non-unit decimal divisors.



1. Rewrite the division expression as a fraction, and divide. The first two have been started for you.

i. $3.6 \div 1.2 = \frac{3.6}{1.2} = \frac{3.6 \times 10}{1.2 \times 10} = \frac{36}{12} = 3$

j. $0.36 \div 0.12 = \frac{0.36}{0.12} = \frac{0.36 \times 100}{0.12 \times 100} = \frac{36}{12} = 3$



4. Two wires, one 17.4 meters long and one 7.5 meters long, were cut into pieces 0.3 meters long. How many such pieces can be made from both wires?

Wire #1: $17.4 \div 0.3 = \frac{17.4}{0.3} = \frac{17.4 \times 10}{0.3 \times 10} = \frac{174}{3} = 58$

Wire #2: $7.5 \div 0.3 = \frac{7.5}{0.3} = \frac{7.5 \times 10}{0.3 \times 10} = \frac{75}{3} = 25$

$$3\overline{)174} \quad -15 \quad 24 \quad -24 \quad 0$$

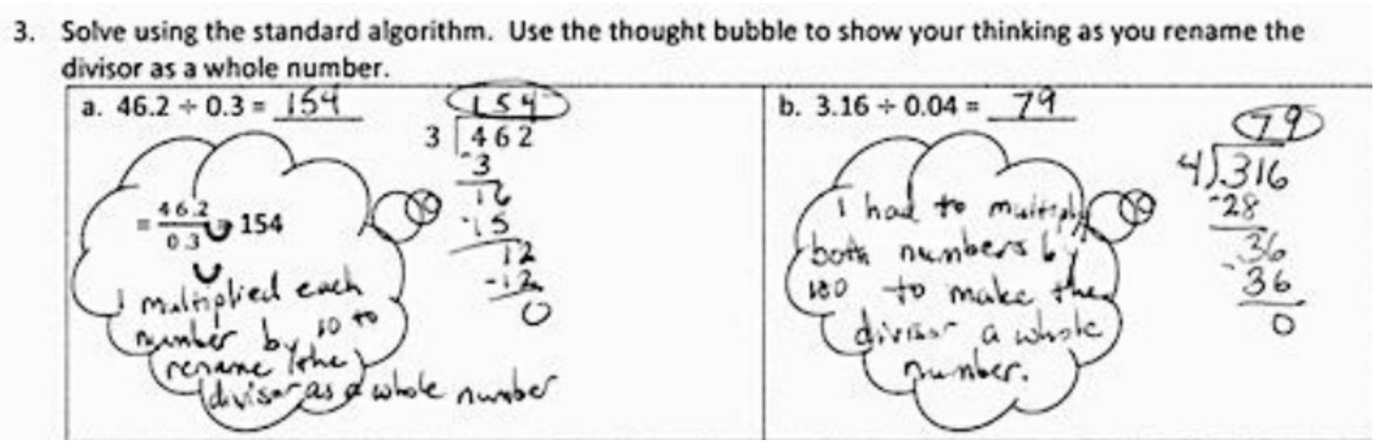
$$3\overline{)75} \quad -6 \quad 15 \quad -15 \quad 0$$

$58 + 25 = 83$

83 pieces can be made from both wires.

Lesson 31

Objective: Divide decimal dividends by non-unit decimal divisors.



3. Solve using the standard algorithm. Use the thought bubble to show your thinking as you rename the divisor as a whole number.

a. 46.2 ÷ 0.3 = 154

$= \frac{46.2}{0.3} = 154$

I multiplied each number by 10 to rename the divisor as a whole number

b. 3.16 ÷ 0.04 = 79

I had to multiply both numbers by 100 to make the divisor a whole number.

Lesson 32

Objective: Interpret and evaluate numerical expressions including the language of scaling and fraction division.

3. Fill in the chart by writing an equivalent numerical expression.

a.	Half as much as the difference between $2\frac{1}{4}$ and $\frac{3}{8}$.	$(2\frac{1}{4} - \frac{3}{8}) \div 2$
b.	The difference between $2\frac{1}{4}$ and $\frac{3}{8}$ divided by 4.	$\frac{(2\frac{1}{4} - \frac{3}{8})}{4}$

7. Evaluate the following expressions.

a. $(9 - 5) \div \frac{1}{3}$

$4 \div \frac{1}{3} =$

12

b. $\frac{5}{3} \times (2 \times \frac{1}{4})$

$\frac{5}{3} \times \frac{2}{4} =$

$\frac{10}{12} = \frac{5}{6}$

Lesson 33

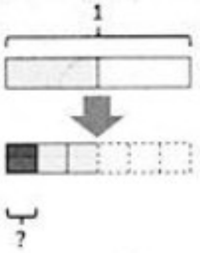
Objective: Create story contexts for numerical expressions and tape diagrams, and solve word problems.

5. Create a story context for the following expression.

$$\frac{1}{3} \times (\$20 - \$3.20)$$

Chris had $20. He bought a bag of pens for $3.20, then spent $\frac{1}{3}$ of the rest on notebooks. How much did chris spend on notebooks?

6. Create a story context about painting a wall for the following tape diagram.



I painted the bottom half of my bedroom wall blue. I then painted $\frac{1}{3}$ of the blue wall red. What fraction of the wall is red?