

Mathematical Thinking Games

Have Fun with Math... and Your Kids!

MATH+PLUS
A C A D E M Y



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Problem Solved

Why Games Matter for Math

Not every parent is well equipped to help their child with the challenges of math at school. Sometimes the math is taught in new ways that are difficult for parents to replicate at home. Other times, parents don't feel confident to "teach" their own child due to their own apprehension about math.

Playing games with your child is the perfect way to avoid these issues and engage your child in mathematical fun that is non-threatening for the parent and the child.

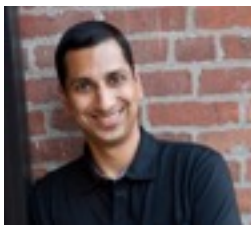
Games allow parents to immerse kids in hands-on mathematical experiences instead of resorting to workbooks or flashcards.

By playing games, you help your child develop mathematical thinking skills naturally without turning math into a chore.

Benefits of Games Include:

- They are fun for the whole family
- They are interactive and collaborative
- They take the pressure out of math and put it into meaningful context.
- They involve problem solving, critical thinking, and estimation, and arithmetic skills

We've collected a handful of games that you can explore with your child. **Have fun!**



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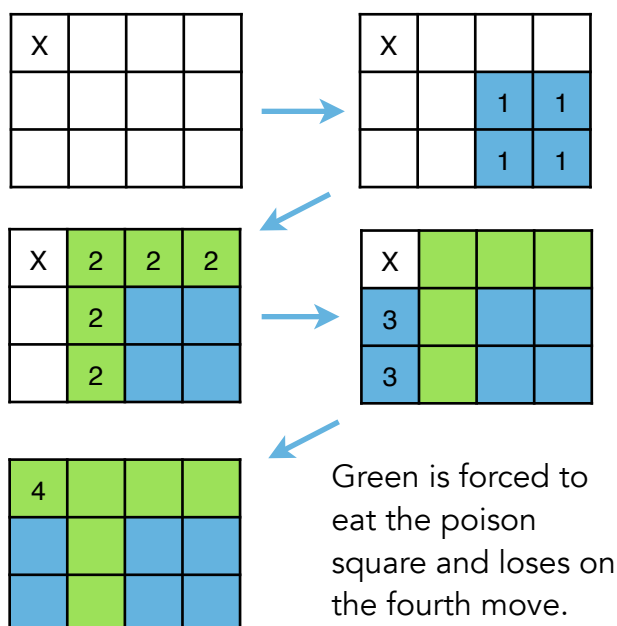
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Chomp

1. Draw a rectangular grid of squares to represent a chocolate bar with many pieces. Mark the top left square with an 'X' — this square is poisoned. Players can choose the size of the starting grid.
2. Players take turns in taking a rectangular bite out of the bottom right corner of the bar, by shading a square, together with all the squares below and/or to the right of it.

The player forced to eat the poison square loses.

Sample 3x4 Game



Tips to Play and Learn

Play several games with a small grid and see if you discover any good strategies.

Can the first player always win? Or will they always lose?

What is the smallest board you can play on 1x1? 1x2? 2x1? How big does the board need to be to create an "interesting" game?

Is there a best first move? If so, what is it?

What is the largest number of moves possible in a particular game?

How do things change if the board has an odd number of rows and columns? both even? one even and the other odd?

Trying changing the rules: The player that gets the last piece wins. How does this change your strategy?

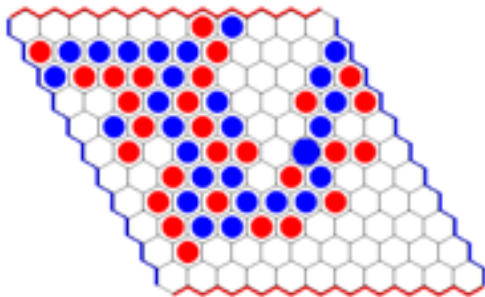
Trying changing the rules: What happens if the 'X' isn't in the top-right corner?

Hex

1. Two players take turns marking a cell with their color on an 11x11 hexagonal grid.
2. One player tries to connect the top and bottom, the other tries to connect from left to right.

First player to make it across wins.

Sample Game



Blue wins. Do you see how?

Tips to Play and Learn

While learning, always voice your strategies so you both can learn. You're collective understanding of the game will increase much faster this way.

Try playing on much smaller boards to help develop your thinking about the game.

Play several games and see if you discover any useful strategies.

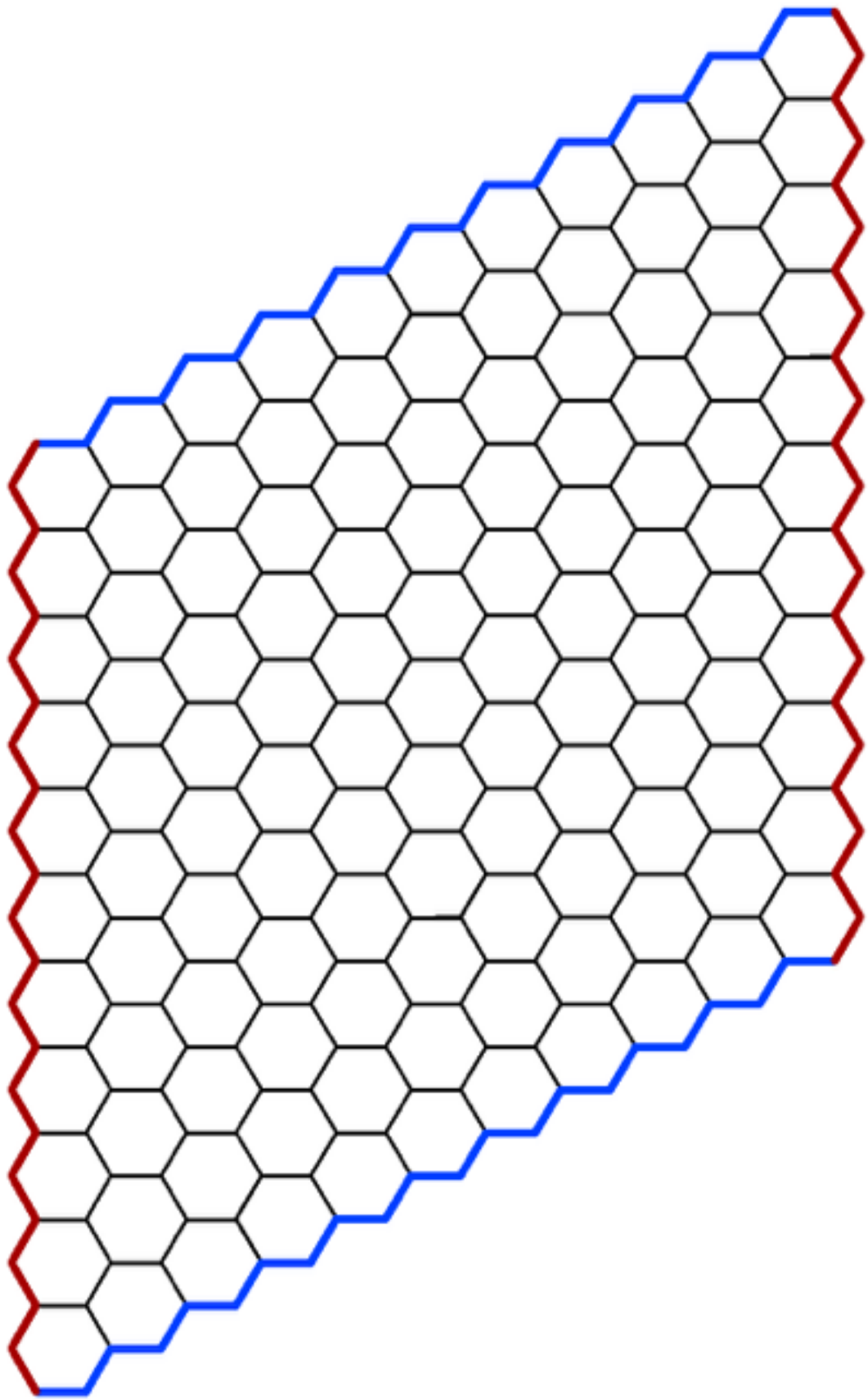
Does the person that goes first have an advantage?

How far away should you place your marker to best block your opponent?

Set up some hypothetical situations and ask your opponent what they would do next.

Place a red marker three rows from the bottom of the board. Figure out if you can stop red from reaching the bottom edge.

Variations: Can you think of ways to modify the game to turn it into a new challenge?

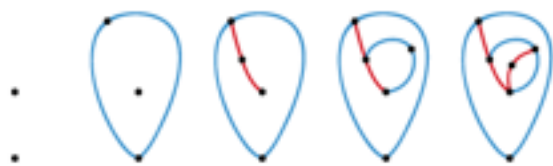


Sprouts

1. Players agree to a fixed number of dots to start the game.
2. Players take turns connecting dots following these rules.
 - a) Draw a curve joining two spots, or a single spot to itself. The curve must not pass through another spot or another line.
 - b) Draw a spot on the new curve.
 - c) No more than three lines can emerge from any spot.

Last player to make a legal move wins.

Sample Game



Starting with two dots. Red wins after 4 moves, because blue has no move.

Tips to Play and Learn

Start with the simplest game, 2 dots.

Does the same player always win? If so, why? If not, why not?

Starting with two dots, what are the most number of moves that can be made?

What is the greatest number of moves?

Sometimes dots get “trapped” and can’t be used even though they have fewer than 3 connecting lines. How can you trap dots? When is it best to trap dots?

Can you start with so many dots that the game will never end? If so, prove it. If not, why not?

Dots that don’t have three lines at the end of the game are called “survivors”.

The bottom dot in the sample game is a survivor. How does the number of survivors impact who wins the game?

Variations: How could you modify this game to make up a new and interesting game of your own?

Break the Code

1. One player, the code-maker, creates a four-digit secret code. The second player, the code-breaker, tries to deduce the secret code.
2. The code-breaker makes a guess, then code-breaker replies with:
 - a) # of digits correct in the right position — “Exact”
 - b) # of digits correct in the wrong position — “Close”

The goal is to break the code in the fewest number of guesses.

If either number has repeated digits, then each digit can only count towards the score once with exacts counted first.

Sample Game

Code: 2574

- Guess 1: 1234 - Reply 1 close, 1 exact
Guess 2: 8567 - Reply: 1 close, 1 exact
Guess 3: 2864 - Reply: 2 exact
Guess 4: 4257 - Reply: 4 close
Guess 5: 2574 - Reply: 4 exact!

Tips to Play and Learn

Note: The game can be simplified for younger players by reducing the number of digits. Also, it's best to start by not allowing repeating digits in the code.

What are the best first guesses?

Is it good or bad if a guess has no close or exact digits?

How many secret codes are possible?

How many guess would yield no close or exact digits for a given code?

Can you explain some strategies you might use to limit your total number of guesses?

Is it useful to guess all the same digit, like 4444?

Do you think there is an optimal strategy for playing the game? If so, what is it?

If you're ready, try playing with a five-digit code!

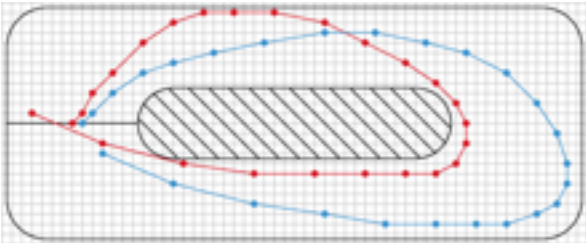
Racetrack

1. Players draw a racetrack on grid paper.
2. Players take turns plotting the position of their racecar following these rules:
 - a) Each car is initially stationary
 - b) The car moves the same direction and distance as the previous move, or it can accelerate or decelerate by one grid point in any direction.

The goal is to finish first.

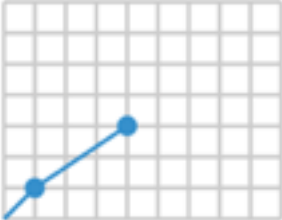
If either number has repeated digits, then each digit can only count towards the score once with exacts counted first.

Sample Game

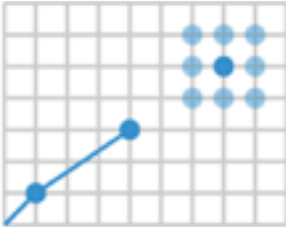


Sample Play

Suppose in the previous move Blue moved three squares right and two squares up:



The next move can be three squares right and two squares up again (dark blue spot), or any of the eight positions around this (light blue spots):



It may take you several tries to complete the race without crashing into a wall (Crashes result in loss of turn and speed dropped to zero again.)

Try all sorts of different shaped tracks!

One-Pile Nim

1. Players agree to a starting number of tokens.
2. Each player may remove one or two tokens on their turn.

The goal is to take the last token.

Sample Game

Start with 7 tokens.

Player 1 takes 1, leaving 6

Player 2 takes 2, leaving 4

Player 1 takes 2, leaving 2

Player 2 takes 2, leaving none.

Player 2 wins!

Tips to Play and Learn

What happens if you leave just one token at the end of your turn?

What happens if you leave just two tokens at the end of your turn?

What happens if you leave just three tokens at the end of your turn?

What about 4 tokens left at the end of your turn?

Can you see a pattern developing?

Are there scenarios where the first player can always win?

Are there scenarios where the first player will always lose (assuming the opponent plays an optimal strategy)?

If you knew you would go first, how many tokens would you like the game to start with?

If you knew you would go second, how many tokens would you like the game to start with?

Variation: Allow players to take one, two, or three tokens on each turn. How does this affect your strategy?

Variation: The last token is poison, so the player that takes the last token loses!.

How does this affect your strategy?

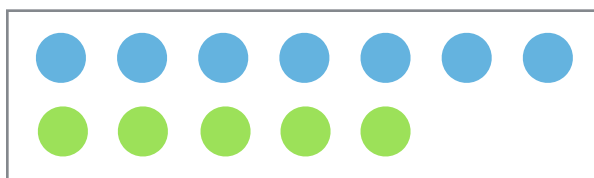
Multi-Pile Nim

1. Players agree to a starting number of tokens and separate them into two or three piles (each pile can have any number of tokens).
2. Each player can remove ANY number of tokens from ONE pile on each turn.

The goal is to take the last token.

Sample Game

Start with 7 tokens in one pile (blue) and 5 tokens in a second pile (green).



Player 1 removes 6 blue tokens then
Player 2 removes 3 green tokens



Player 1 removes 1 green token then
Player 2 removes 1 green token. Player 1
wins by taking the final (blue) token.

Tips to Play and Learn

What happens if you leave just one token at the end of your turn?

What happens if you remove all the tokens from a pile in a two-pile game?

Try mimicking your opponents moves and see what happens.

Start with two piles and a small number of tokens in each pile to develop a sense for the best strategies in a two-pile game.

Try three piles and a small number of tokens in each pile to develop a sense for the best strategies in a three-pile game.

Is it easier to analyze the game by working backwards from the end back towards the beginning of the game?

Variation: In a two-pile game, players can take any amount from one pile or the SAME amount from both piles.

Shut the Box

1. Write the numbers 1 - 10 on a paper.
2. Roll two 6-sided dice and compute their sum. Cross off the sum or any two numbers that make the sum from your list of numbers 1 - 10.
3. Repeat step 2 until no options remain.
4. Add remaining numbers from the list together for your score. Then the next player takes their turn.

The goal is get the lowest score.

Sample Game

Roll 3 & 4 = 7. Cross off 5 & 2.

[1 2 3 4 5 6 7 8 9 10]

Roll 5 & 5 = 10. Cross off 10.

[1 2 3 4 5 6 7 8 9 10]

Roll 6 & 2 = 8. Cross off 7 & 1.

[4 2 3 4 5 6 7 8 9 10]

Roll 3 & 3 = 6. Cross off 6.

[4 2 3 4 5 6 7 8 9 10]

Roll 4 & 2 = 6. There are no pairs that sum to 6 remaining. Score: $3 + 4 + 8 + 9 = 24$.

Tips to Play and Learn

Is it best to cross off the largest numbers or the smallest numbers first?

Would you rather roll large sums or small ones?

Would you rather have even sums or odd ones?

If you roll a sum of 8, how many possible pairs can you cross off?

Variations: The best part of this game is the flexibility it provides to create new rules.

- Change the starting numbers.
- Add dice or use 10- or 12-sided dice
- Allow subtraction or multiplication of dice values. You might need to change the set of starting numbers too.
- Let your child develop their own versions to play. Experiment!